One True Logic?*

Gillian Russell

April 16, 2007

Logic is the study of validity and validity is a property of arguments. For my purposes here it will be sufficient to think of arguments as pairs of sets and conclusions: the first members of the pair is the set of the argument's premises and the second member is its conclusion. An argument is valid just in case it is truth-preserving, that is, if and only if, whenever all the members of the premise-set are true, so the conclusion is true as well.

The domain of logic, then, might be thought of as a great collection of arguments, divided into two exclusive and exhaustive subcollections, the valid and the invalid, the good and the bad, and the task of the logician as that of dividing one from t'other. But working with that conception, one might be puzzled by a fact about research in logic as it is actually pursued: we speak not of *logic*, but of *logics*—classical, modal, relevant, tense, abelian, intuitionistic, counterfactual, paraconsistent—and of arguments which are valid or invalid *in* particular logics. This might suggest that the territory which logic studies is not to be partitioned into two subsets—the valid and the invalid—but rather into many pairs of subsets: the valid and the invalid relative to classical truth-functional logic, the valid and the invalid relative to the modal logic $S5$, the valid and the invalid relative to second order intuitionistic modal tense logic with quantifiers, identity, names, descriptions and functions etc.

But to some extent that pluralism is an illusion, and we can make sense of logic as it is practiced without giving up on the idea of a single correct partition. Classical truth-functional logic and $S5$, for example, need not be thought of as disagreeing on the sets of valid and invalid arguments, but can be considered to be two different attempts to define them. $S5$, being more powerful, goes further and captures more of the target set—but truth-functional classical logic has its own advantages and might sometimes be preferred for stylistic or even epistemological reasons. Thus pairs of logics where one is a sublogic of the other need not be thought of as rivals.[1]

Even where logics were originally developed as rivals, i.e., in the case of intuitionistic logic, we have the option of resconstruing their purpose, for example

*I'm very grateful to participants in the 2007 Mathematical Methods in Philosophy Conference at Banff for their comments and suggestions, especially JC Beall, Patricia Blanchette, Hannes Leitgeb, Marcus Kracht, Augustín Rayo, Greg Restall, and Timothy Williamson.

[1] Of course, one may still accept the sublogic without accepting the superlogic if one considers the superlogic to have overshot, as Quine does in the case of modal extensions of classical logic. [Quine, 1966], [Quine, 1953].

as an offering to a logical sub-project: that of isolating the *constructively valid* arguments from the rest.

Finally, one might also hold that the appearance of a multiplicity of different sets of valid arguments arises from the fact that so many researchers have failed in their tasks. From the fact that there are a multiplicity of different views on what the right answer to a question is, it does not follow that there is more than one right answer to the question; perhaps some sophisticated enhancement of classical logic captures the one true set of valid arguments, and all other contenders are best understood as creative but deceptive failures.

Yet, as Beall and Restall argue in their book [Beall and Restall, 2006] there may be a way to make sense of the idea that there *is* more than one set of valid arguments.

1 Logical Pluralism

Here is a simple idea: suppose we say that an argument is valid just in case *in every possible situation in which all the premises are true, the conclusion is true.* There are, of course, many different conceptions of necessity: logical, conceptual, metaphysical, epistemic, nomological, physical etc. and we might generate even more fine-grained conceptions by including world-relative notions of possibility, and placing different restrictions (reflexivity, transitivity, symmetry) on the accessibility relation.

Some of these conceptions of necessity are stricter than others: for example, it seems natural to think of metaphysical necessity as stricter than nomological necessity, since the set of nomologically possible situations is a proper subset of the set of metaphysically possible situations. In order to be metaphysically necessary, a claim has to be true across all the nomologically possible situations, plus some. Because of this, It is harder to be metaphysically necessary than nomologically necessary, and so metaphysical necessity is a stricter kind of necessity.

But other conceptions of necessity may be incommensurable in terms of strictness, for example, metaphysical necessity where the accessibility relation is reflexive and transitive and metaphysical necessity where the accessibility relation is reflexive and symmetric. On the former, we might think that $\Box P \supset \Box\Box P$ is necessary, but on the latter it is not, since $\Box P$ might be true at all worlds accessible from a world Γ without $\Box P$ being true at any of those worlds. But then, we might think that $\Diamond P \supset \Box\Diamond P$ is necessary when we conceive of the accessibility relation as reflexive and symmetric but not when we consider the relation as reflexive and transitive. So there are many different kinds of necessity and possibility, some of which are stricter than others, but not all of which are commensurable in terms of strictness.

Now if the 'possible' in our definition of validity is ambiguous between different conceptions of possibility, and there is no reason to think one interpretation of the expression privelaged in our discussion, we will find that the expression 'valid' is similarly ambiguous. Small wonder then, that on one disambiguation

it picks out one set of arguments, and on another a different one. If is true, then it might be that one logic provides the right answer to a question of an argument's validity *on one disambiguation* of 'validity' and another provides it for a different disambiguation.

Beall and Restall's thesis in [Beall and Restall, 2006] is both more general and more sophisticated than this simple idea. It is more general in that they decline to limit the situations quantified over in their definition to *possible* situations. Their definition is as follows:

[**Validity** (Beall and Restall)] A valid argument is one whose conclusion is true in every case in which all its premises are true. [Beall and Restall, 2006, 23]

It is more sophisticated in that they motivate their view that 'validity' is ambiguous by suggesting that the intuitive, informal notion of validity is somewhat vague, and may be precisified (or explicated) in a number of different ways. They write that "the pretheoretic notion of logical consequence is not formally defined and it does not have sharp edges" and they represent Tarski as having suggested one precisification:

[**The Restricted Thesis (RTT)**] "The sentence X follows logically from the sentences of the class K if and only if every model of the class K is also a model of X."

The models quantified over in this restricted thesis are the usual Tarski models for classical first-order predicate logic with quantifiers, but Beall and Restall hold that a generalised version of that thesis can be made more precise in a variety of ways, by taking a variety of different things to count as *cases*:

[**Generalised Tarski Thesis (GTT)**] An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion.

Their view, strictly, is the view that there are at least two different kinds of case which can be used to generate admissible restricted versions of the generalised Tarski thesis. In particular, another kind of case, in addition to the Tarski models, (which support a classical conception of validity) include *situations*, which are rather like possible worlds except that they need not be complete. This conception of case supports a relevant account of consequence, according to which the classically valid argument form $A \vDash B \vee \neg B$ is not valid.

2 Bearers of Truth

Having discovered one ambiguity in the definition of validity, one might wonder whether there are others, and whether they might also generate different logics. If there were, these logics might turn out to be logics with which we are already familiar, or they might turn out to be new things entirely.

One place where we can find an ambiguity is in the notion of an *argument.* I have suggested that we think of an argument as a pair consisting of a set of premises and a conclusion. Consider this one:

All men are mortal.
Socrates is a man.

Socrates is mortal.

What kind of thing are the premises and the conclusion in this argument? They are clearly truth bearers of some kind, which narrows the options considerably, but leaves at least the following as candidates: sentences, propositions, characters, statements, utterances, occurrences of sentences, beliefs and judgements.

The question of what the constituents of arguments are is harder than it seems. Logicians do not spend a lot of time on it, any more than mathematicians spend a lot of time worrying about what numbers are. There are at least three good reasons for this. First, many logic books are written for beginners, and few shy from from fibbing to beginners in the name of clarity. Why spend space and your student's attention span working out what a premise is when something like "declarative sentence" will do to be going on with, so that the author can introduce more important things, like the definitions of validity and soundness.

Second, many logic books and articles are written for *non*-beginners. Authors of such textbooks anticipate that their audience is not really interested in the metaphysics of arguments. They want to see proofs, model theory, constructions, results and the like, and they might quickly become impatient with a discussion of something that can seem both elementary and pedantic.

And finally, another explanation of the fact that logicians don't spend much time on this issue might be that they think that *it just doesn't matter.* Not only in the sense that it is unimportant to World Peace and won't buy you groceries, but also in the more salient sense that it won't make any difference to the logic (the constructions, the results etc.)

One might be encouraged in this last view if one holds the influential (and, I think, correct) view that propositions are the primary bearers of truth, where that is to say that all other truth bearers inherit their truth-values from them. For example, on this view, the reason that the sentence 'snow is white' is true is that it says something true, that is, it expresses the true proposition *that snow is white.* Had it expressed a false proposition, then the sentence would have been false. Similarly, the reason my belief/statement/claim/utterance/declaration is true is that it is a belief/statement/claim/utterance/declaration *that snow is white* and that is a true proposition.

Now, validity, of course, is preservation of truth, and so one might expect that just as truth is inherited by other truth-bearers from the propositions they express, so validity is inherited by arguments when the truth-bearers they contain express propositions across which truth is preserved.

Let me try to make this extension of the idea that propositions are the primary truth-bearers clearer. Let B be any non-propositional truth-bearer. Then we might expect that in general, an argument:

(1) $\dfrac{\begin{array}{l}B_1\\B_2\end{array}}{B_3}$ is valid iff $\dfrac{\begin{array}{l}P_1\\P_2\end{array}}{P_3}$ is valid, where B_i expresses P_i.

The thought that it just doesn't matter what kind of truth-bearer the premises and conclusions of arguments are could be forced home as follows: this is not an interesting question because we can take the constituents of arguments to be just about *any* kind of truth-bearer, and our choice will have no effect on our logic: other kinds of arguments inherit their validity from the propositional case.

I hold that in general this thought is wrong. For some values of B_i and P_i, where B_i expresses P_i, $B_1...B_n \vDash B_m$ even though $P_1...P_n \nvDash P_m$. Moreover, for other values of B_i and P_i, where B_i expresses P_i, $B_1...B_n \nvDash B_m$ even though $P_1...P_n \vDash P_m$. It will follow that the equivalence fails in both directions.

The simplest way to set the counterexamples up is to adopt a Russellian view of propositions, of the sort encouraged in [Kaplan, 1989]. On the view I have in mind, names are directly referential, so that their only contribution to the proposition expressed by a sentence containing them is their referent. If we represent propositions as sequences of objects and properties, then we can represent the proposition expressed by 'Hesperus is Hesperus' as an ordered triple as follows:

(2) Hesperus is Hesperus
$\langle =, \circledcirc, \circledcirc \rangle$

where '$\circledcirc$' is a name of Hesperus and '$=$' expresses identity. This is the proposition that Hesperus is identical to Hesperus. Since 'Phosphorous' refers to the same object as 'Hesperus', it makes the same contribution to the proposition expressed by a sentence containing it, and hence the proposition expressed by 'Hesperus is Phosphorus' is just that expressed by 'Hesperus is Hesperus':

(3) Hesperus is Phosphorus.
$\langle =, \circledcirc, \circledcirc \rangle$

Now let P_1 be the proposition $\langle =, \circledcirc, \circledcirc \rangle$. Since validity is a matter of truth-preservation, and, necessarily, for any B_i B_i is true iff B_i is true, we know that that '$\vDash$' is reflexive, and hence $P_1 \vDash P_1$.

Yet, at least as we normally construe it,

(4) $\dfrac{\text{Hesperus is Hesperus.}}{\text{Hesperus is Phosphorus}}$

is *not valid*. As I have argued, it is not terribly clear what we normally take the premises and conclusion to consist of in (4), but whatever they are—sentences, characters, perhaps utterances—call the premise B_1 and the conclusion B_2. Now we have our counterexample, for B_1 expresses P_1 and B_2 expresses P_1, yet $P_1 \vDash P_1$ and $B_1 \nvDash B_1$

Not everyone accepts that 'Hesperus is Hesperus' and 'Hesperus is Phosphorus' express the same proposition, of course, but there will be less contentious counterexamples to come and moreover, the equivalence thesis was supposed to be a triviality that allowed us to ignore a complexity in the study of logic—nothing was said about it requiring the rejection of a currently popular view in the philosophy of language.

Even if they are not direct reference theorists about about names, many philosophers hold that different sentences within the same language may say the same thing, that is, that different sentences may express the same proposition. Common examples of this phenomenon include paraphrases of sentences containing indexicals using other indexical sentences, or sentences that are free of indexicals. For example what I say by uttering 'I am in Banff', you may say by uttering (in the relevant context) 'You are in Banff' or 'The author of 'One True Logic?' is in Banff.' Since neither (5) nor (6)

(5) $\dfrac{\text{I am in Banff.}}{\text{You are in Banff.}}$

(6) $\dfrac{\text{I am in Banff.}}{\text{The author of 'One True Logic?' is in Banff.}}$

are valid as normally construed, we have two more examples of invalid arguments whose conclusions express propositions which do follow validly from the propositions expressed by the premises (again, by the reflexivity of '$\models$'.) Hence the validity of the propositional argument has failed to be inherited by the argument as we normally conceive of it.

Finally, consider:

(7) $\dfrac{}{\text{I am here now.}}$

This argument is *valid*, since 'I am here now' is a logical truth of Kaplan's logic LD [Kaplan, 1989], yet the sentence did not inherit its validity from the proposition it expresses. There are two ways to see this. The first is that the proposition it expresses (say, as said by me as I am giving this paper) is *not* itself valid; I needn't have been here now, I might have had a terrible accident cross-country skiing yesterday and have been spending this afternoon in the local hospital. And secondly, the sentence expresses *different* propositions in different contexts, none of which need be valid. What is special about this sentence is not that the proposition it expresses cannot be false, but rather that it cannot express a false proposition. It is something special about how the proposition expressed by this sentence is determined in a context that makes it a logical truth, not something about the proposition it happens to express in this context.

In this last instance we have a counterexample to the *other* direction of the biconditional. Let 'I am here now' $= B_1$ and P_1 be the proposition it expresses. Then B_1 is valid even though B_1 expresses P_1 and P_1 is not valid. Hence the truth-bearer did not inherit its validity from the proposition it expresses.

The Quine/Goldfarb View

Before we move on, I should note that there's a notable exception to my claim about logicians being uninterested in this issue, and their not going into it in their writings. It does not apply to Quine and books by Quine, and the trait seems to be heritable, since the same is true of some of Quine successors and *their* books, such as Warren Goldfarb and his [Goldfarb, 2003]. So perhaps another reason other logicians say little or nothing about it is that they think that the work has already been done by Quine.

According to the Quine/Goldfarb story, arguments are composed of *statements*. A statement is a special kind of sentence, namely one which has a truth-value. For example:

(8) All men are mortal.

(9) Snow is white and grass is green.

(10) If all men are mortal, then snow is white.

(11) It is not the case that it is not the case that ants have a system of slavery.

are all statements, whereas

(12) Hello.

(13) Where are you going?

(14) I am going to the hot springs.

(15) He is going cross-country skiing.

are not. The first is obviously, not truth-evaluable, nor the second, since it is a question. The third contains an indexical and the fourth a demonstrative and as a result they can take different truth-values in different contexts of utterance, and so, since in logic we only consider the sentence type, neither has a proper truth-value independently context, which is what would be needed for it to figure in an argument. This leads Quine to suggest that, even though arguments consist of statements, it is, strictly speaking, only "individual events of statement utterance" that are truth-evaluable [Quine, 1950, 1] and Goldfarb suggests that we either paraphrase linguistic forms containing indexicals and demonstratives into proper statements, or "avoid this tedium by imagining the sentences of our examples...to have been uttered by a single speaker, at a single time, in a conversational setting that uniformly resolves any ambiguities." [Goldfarb, 2003]

But the development of indexical logics has shown that this Quinean answer to my question is inadequate. If sentences containing indexicals cannot form the premises or conclusions of arguments, (unless we assume them to have been uttered in a single context), what are we to make of the following arguments, all of which are both intuitively valid—even when not relativised to a context—and valid in Kaplan's Logic of Demonstratives (LD)? [Kaplan, 1989]

(16) $\dfrac{\text{I am in Banff.}}{\text{Someone is in Banff.}}$

(17) $\dfrac{\text{Snow is white.}}{\text{Actually, now, snow is white.}}$

(18) $\dfrac{\quad}{\text{I am here now.}}$

The strict version of the Quinean view entails that (16)–(17) are not really arguments, since they contain sentences containing indeixicals. Since they are not only arguments, but valid arguments, the strict version of the Quinean view is inadequate.

Goldfarb suggests that we allow such forms to count as arguments so long as we relativise them to particular contexts, so that the truth-value of any sentence containing an indexical is settled. Thus on this view we cannot settle once and for all the validity of the arguments above, but only the validity relative to particular contexts. But this is wrong. All the arguments above are valid precisely because in *any* context in which the premises are true, the conclusion is true.

3 Truth and Validity

In the rest of this paper I will investigate the following three questions: i) what kind of answer to the question "what are arguments made of?" would make most sense of how we actually do logic? ii) could arguments be made of something else, and would logic look different if we investigated arguments so composed? iii) if so, would the upshot be a different kind of logical pluralism?

3.1 Making Sense of Logicians

What should we take the components of arguments to be if we are to make the most sense of ordinary practice in logic? The most obvious candidate answers are sentences, statements, utterances, propositions and beliefs, as well as combinations of these; just as Quine talked of sentences with a truth-value, so we might speak of sentences paired with propositions, or sentences with characters.

I will begin by eliminating some of the weaker candidates. First up: beliefs. "Belief" is an ambiguous expression. In one sense, a belief is a content—a proposition—and in another it is something in people's heads that psychologists study. Following Frege, I think that logic is not really concerned with belief in the psychological sense. [Frege, 1956] Modus Tollens is truth-preserving regardless of any thinking agent's tendency to take it to be so, or fail to take it to be so, and an argument can be valid even if its premises are something that no-one has ever believed, for example:

(19)

The smallest grain of sand Tarski ever saw was 100 years old, but not heavy.

The smallest grain of sand Tarski ever saw was 100 years old.

But a belief can also be a proposition, which is why I can say things like:

(20) His belief is that Mars is dry.

(21) Peter believes that too, but it's false.

(22) You might think that, but it isn't what he believes.

On this conception, a belief is not something in a particular person's head, but something that more than one person can share, something that may be true or false and that can be referred to in English using a 'that'-clause.

Next target: propositions. Despite the wide-spread practice of referring to truth-functional logic as "propositional logic", when we attribute validity to an argument, we don't normally take that argument to consist of propositions, otherwise we would take these arguments to be valid:

(23)
Hesperus is Hesperus.

Hesperus is Phosphorus

(24)
I am in Banff.

Gillian Russell is in Banff.

We do not take these arguments to be valid, in fact we do not even take the *posibility* that the premises and the conclusions express the same proposition to threaten the judgement that the arguments are invalid. So the best way to construe our ordinary practice is not to construe us as studying the validity of arguments composed of propositions.

Nor are we usually studying the validity of arguments composed of *sentences*, when these are understood as uninterpreted types or tokens, since uninterpreted sentences are just marks on the page, sounds or signs in the air, or types of these, and no such thing is a truth-bearer until it has an interpretation.

Yet perhaps the constituents of arguments are, along the lines Quine suggested, sentences with a particular kind of interpretation. We might think of such objects as ordered pairs of (syntactically defined) sentence types and meanings, like this:

$$\langle S, M \rangle$$

Now we need to ask what kind of thing M is. Is it the proposition the sentence expresses? The sentence's character? Or something else?

Suppose we take the constituents of arguments to be ordered pairs of sentences and the propositions they express. This view has the advantage that it makes sense of how the validity can be relative to a language, since the same

sentences (construed syntactically) may express different propositions.

$$\frac{\langle S_1, P_1\rangle \quad \langle S_2, P_2\rangle}{\langle S_3, P_3\rangle} \qquad\qquad \frac{\langle S_1, P_1^*\rangle \quad \langle S_2, P_2^*\rangle}{\langle S_3, P_3^*\rangle}$$

However, given that sentences inherit their truth-values from the propositions they express, it seems that, if arguments really were composed of pairs of sentences and the propositions they expressed, the following argument would be valid, since it really is impossible for the premises to be true and the conclusion false, given that the premises and the conclusion express the same proposition.

(25) $\dfrac{\text{Hesperus is bright.}}{\text{Phosphorus is bright.}}$

On the view we are considering, we can represent this argument as follows:

$$\frac{\langle S_1, P_1\rangle}{\langle S_2, P_1\rangle}$$

This time we have had to appeal to more than the reflexivity of '$\models$' to show that the argument is valid, since the premises and the conclusion are different, but the validity of the argument follows from the assumptions that the two sentences express the same proposition and that the truth-value of an interpreted sentence $\langle S_i, P_i\rangle$ is the truth-value P_i. The *reason* that (25) is not valid is not that either of these assumptions fail, but rather that the components of the argument are *not* sentences paired with the propositions they express.

On the view that identifies the constituents with sentence-proposition pairs, it also seems hard to explain the validity of this argument:

(26) $\dfrac{}{\text{I am here now.}}$

Why should 'I am here now' be a logical truth when other sentences which express the same proposition are not? That is, why is (26) valid when (27) is not?

(27) $\dfrac{}{\text{Gillian Russell is in Banff now.}}$

This last difficulty suggests a different approach. 'I am here now' and 'Gillian Russell is in Banff now' may express the same proposition in some contexts, but their *characters*, in the sense of [Kaplan, 1989] are different. So a promising suggestion would be to take the components of arguments to be sentences paired with their *characters*:

$$\langle S_i, C_i\rangle$$

Character is meant to be the component of the meaning of an expression that determines its content relative to different contexts, and which competent speakers of the language know (at least tacitly.) For example, in the case of an indexical like 'I', the character might be the rule that 'I' always takes the agent (i.e. the writer, speaker or signer) of the context as its content. On Kaplan's view, speakers use this rule to work out which proposition is expressed by utterances containing the expression in different contexts. Nevertheless utterances of 'I am here now' never express the same proposition as utterances of 'The agent of the context is here now', since these propositions can take different truth-values relative to the same possible world.

Within Kaplan's model theory, character is represented as a function from contexts (quadruples of agent, place, time and possible world) to contents, where contents are (as usual) represented as functions from possible worlds to extensions. Thus characters tell you what content an expression takes relative to different contexts. What is special about the character of a sentence like 'I am here now' is that it always maps the context to a proposition that is true in that context, so that—although the sentence takes different contents in different contexts—it expresses a true content in every context.

As anticipated, this approach fits well with our examples containing indexicals and the validity of (26) is no longer in tension with the invalidity of (27). Since the sentences have different characters, there is no reason to expect one to be valid if the other is.

It might seem surprising that we need such a recent technical device in order to make sense of the way we actually treat validity, but, in fact, further examples show that sentences with characters will not do either. For, in Kaplan's formal system at least, names have constant characters and in particular the character of any name is a constant function from contexts to its referent. It follows that 'Hesperus' and 'Phosphorus' have the same character, and hence that argument (25) can be represented as follows:

$$\frac{\langle S_1, C_1\rangle}{\langle S_2, C_1\rangle}$$

And moreover

(28) $\dfrac{\text{Hesperus is bright.}}{\text{Hesperus is bright.}}$

can be represented the same way. Yet (28) is valid and (25) is not. Hence the components of arguments cannot be sentence-character pairs either.

I should like to distinguish my argument here from a related, fallacious one. I am *not* arguing that if arguments were composed of sentence-character pairs then (28) would have to have the same status as (25) *on the grounds that they would be the* same *argument*—they are not the same argument; the conclusion of (25) is different from that of (28) because the sentential component is different. Rather, I am arguing that if arguments were composed of sentence-character pairs then (28) would have to have the same status as (25) because the

sentence-character pair *gets its truth-value* through its character. If there were no character component to the pair, then the pair would not be a truth-bearer at all. Given that the conclusions of (28) and (25) have the same character, they should get the same truth-values in the same situations. So how can the validity come apart? It seems that sentences must also be associated with something even more minimal than character, something that differs between expressions like 'Hesperus' and 'Phosphorus'.

This leaves us with a puzzle. Arguments composed of sentences alone—syntactically construed—are not the objects of validity, because they are not composed of truth-bearers. Arguments composed of meanings alone—truth-values, intensions, or characters—are not the objects of validity, because different sentences can have the same meanings, and an argument composed of a sentence expressing one meaning (as the premise) and a different sentence expressing the same meaning (as the conclusion) need not be valid (for example, (28). It seems that both mere syntactic differences (using a different but synonymous name) and differences in meaning alone (same sentences with different meanings) was enough to make the difference between a valid and an invalid argument. If both meaning and the exact words used are effect validity, that suggests that the objects of validity are composed of combinations of the two: certain sets of words with certain meanings. But we have seen that sentence-proposition pairs are not fine-grained enough for dealing with indexicals and names, and even sentence-character pairs are not fine-grained enough for dealing with names. It seems that arguments must be constructed from something even more new-fangled and unfamiliar.

3.2 Could things be otherwise?

Suppose that in order to make sense of our actual characterisations of validity, we have to take ourselves to be talking about arguments composed of a certain kind of truth-bearer. That leaves open the question of whether we could do things differently. Is there room for, say, a logic of propositions? Or a logic of statements?

There is. If what is important for validity is that in all cases where the premises are true, the conclusion is true, then it makes sense to talk of validity for *any* kind of truth-bearer. Though it may be that we *normally* only call arguments composed of a certain kind of truth-bearer valid, the notion will be naturally extendable to arguments composed of other things.

Would such an extension be philosophically interesting? That might depend on affirmative answers to two questions. First, would the logic of other truth-bearers be different from the logic we normally do? And second, would it have a different use?[2]

We have already see evidence that the answer to the first question is 'yes'. If we take the following three arguments to be composed of propositions alone, then the first is valid and the second and third are not.

[2]I'm grateful to Agustín Rayo for stressing the importance of this second question.

(29) Hesperus is bright.

Phosphorus is bright.

(30) ---
Gillian Russell is in Banff now.

(31) ---
I am here now.

But if we take them to be composed of sentence and character pairs, then the first and third are valid. Finally if we take them them the way we normally take them, then only the third is valid.

We can make a similar points with the help of an example that Tim Williamson suggested to me:[3]

It is now exactly 8pm.

So it is now not exactly 8pm.

This might be thought to be valid if we take the constituents to be concrete utterances which necessarily have extension in time, but not if we take them to be what Kaplan called *occurrences* of sentences in contexts, where the sentences might be assessed with respect to the same time.

The second question was whether the different logics might be useful for different purposes. This question strikes me as important, but I don't yet know what to say about it. I'd be particular interested in any suggestions that participants in the Second Online Philosophy Conference might have.

3.3 One true logic?

Is the view I have just presented really a form of logical pluralism? Here is one reason to think *not*: Genuine logical pluralism would be committed to there being at least two opposing, but equally correct, answers to the question of whether a single argument is valid. But at first sight, it does not look as if the view I have presented is committed to that. One may indeed, according to the view, ask of the following argument

(32) Gillian Russell is in Banff.

I am in Banff.

as it is presented on the page, whether it is valid or not, and receive two different and equally correct answers. The first might say that the argument is valid, since its premise and conclusion are identical propositions and logical consequence is a reflexive relation, and the second might say (as we normally do) that the argument is not valid, since there are contexts of utterance with respect to

[3]In conversation at the *Mathematical Methods in Philosophy Conference* at Banff International Research Station for Mathematical Innovation, February 2007.

which the sentence-character pair which is the premise is true, and the sentence-character pair which is the conclusion is false; a counter-example would be the context in which Kenny is the agent of the context. But this is not yet full-blown logical pluralism, since the only reason there were two answers to the question was that it was unclear which argument the question was about. Once we disambiguated the question, there remained only the single answer (at least if we bracket the Beall-Restall view for the moment.)

One can think about it differently. If one simply stipulates that arguments are made up of sentences, syntactically construed, then one might say that there is a single argument which is unambiguously picked out in the question above, but that that argument is valid, or invalid, relative to different interpretations, or even, less platitudinously, the question of its validity depends on the depth of the interpretation intended. Assign mere characters to the sentences, and it is possible for the premises to be true and the conclusion false, so the argument is not valid. Assign propositions to them (relative to the context in which this paper was presented) and that is no longer possible, and so the argument is valid. That looks like a stripe of logical pluralism.

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